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Discussion

# Comment on "Fertility, social mobility, and long run inequality" by Juan Carlos Cordoba, Xiying Liu and Marla Ripoll

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## 1. Introduction

Intergenerational persistence in economic outcomes is a well-documented observation. Parents' earnings are a major determinant of their offsprings' earnings and the same holds for measures such as wealth, schooling, etc. Altruism towards children is the traditional explanation for this observation. When parents are altruistic towards their children, persistence in wealth occurs through bequests or inter-vivo transfers while persistence in earnings occurs through human capital and schooling investments.[1]

Standard theories of intergenerational persistence often take as given the assumption that family size is exogenous. Theoretically, it is plausible to assume that parents' wealth or earnings affect their choice of family size. This can have significant implications for intergenerational persistence. In fact, once endogeneity of family size is taken into account, intergenerational transmission can additionally occur through family size, i.e., wealthier parents can have larger family size and not necessarily better fortunes for each off-spring.

The traditional framework for intergenerational analysis of endogenous fertility, the model in Barro and Becker (1989), has a stark implication regarding this composition. It implies that intergenerational transmission occurs solely through family size, i.e., per capita bequest left for children is independent of parent's fortune. The paper by Cordoba et al. (2015) is an attempt to reconcile intergenerational persistence with altruism and endogenous fertility. As they illustrate, the main additions to the standard framework are exponential discounting with respect to fertility, discrete number of children, and diminishing costs of children.

In this note, I discuss the failure of the benchmark Barro and Becker (1989) model (BB henceforth) in generating intergenerational transmission and Cordoba et al. (2015) way of fixing this issue. I, then, discuss alternative routes in reconciling the BB model with intergenerational transmission.

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[1] Loury (1981) is among the first to formalize this idea and provide a framework that connects the distribution of parental wealth to children's wealth and earnings that can be used for distributional policy analysis.

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## 2. Intergenerational transmission in the Barro–Becker model

The Barro and Becker (1989) and its extension in Alvarez (1999) is the main workhorse for the analysis of determinants of fertility over time and in the cross-section. In its basic version, it is consisted of one-period lived individuals who decide how to save, i.e., leave bequests for their descendants and decide on the size of their family. The trade-offs involved in these decisions are captured by the following simple optimization problem:

$$\max_{c,B',n} \frac{c^{1-\sigma}}{1-\sigma} + \beta n^{\epsilon} V\left(B'/n;\omega\right) \tag{1}$$

subject to

$$c+\lambda(\omega)n+B' = \omega+(1+r)b$$

where in the above, $c$ is life-time consumption, $n$ is family size, $B'$ is total intergenerational transfer to children (which for simplicity I will refer to as bequests), $\omega$ is life-time labor income, $b$ is total assets, and $\lambda(\omega)$ is the per-child cost of child rearing. Furthermore, preferences towards descendants is captured by the function $n^{\epsilon}V(B'/n;\omega)$ where $V(\cdot;\cdot)$ can be the utility of the representative descendant – standard altruism – or any function that captures the utility that the individual receives from leaving bequests – joy of giving.

The above formulation arises in various models of endogenous fertility. In the original framework of Barro and Becker (1989), (1) is the problem that the representative household solves while in the model of Alvarez (1999) where generations are subject to idiosyncratic shocks, $\omega$ evolves stochastically.

Evidently, the problem in (1) is a standard consumer problem. A key property of the objective in (1) is that preferences are homothetic in $(n,B')$. Mathematically, this means that multiplying $B'$ and $n$ by a constant factor does not change the preference ordering of the individual. Given these preferences the marginal rate of substitution between $n$ and $B'$ depends only on per capita bequests while the marginal rate of transformation is constant.

Intuitively, this property is equivalent to the idea that increasing wealth does not change the ratio of spending on bequests to that on children. As a result, an increase in the individual's wealth $b$, leads to an increase in the family size as well as total bequests by a proportional amount. In other words, wealth-expansion paths are linear in $b$. This feature is depicted in Fig. 1 below. An increase in $b$ keeps per-capita wealth constant.

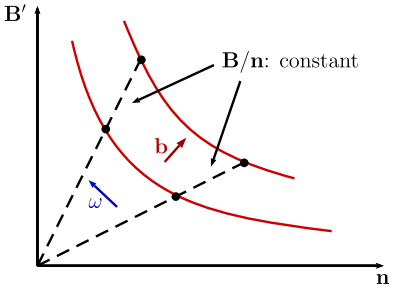


**Fig. 1.** Indifference curves and optimal choices in $(n,B')$ space in the canonical BB model.

An important implication of this feature is that per capita wealth of children are independent of parental wealth. That is, intergenerational transmission does not occur through wealth. Note that as it is illustrated in Fig. 1, children's wealth is correlated with parental income since cost of child-rearing depends (typically positively) on parental income. However, after controlling for income, there is no intergenerational persistence in wealth. This result is arguably in contrast with the rather large intergenerational elasticity of wealth – estimates by Mulligan (1997) and Cordoba et al. (2015) put this elasticity close to 0.5 (This number is higher than estimates for earnings which is close to 0.4).

It is worth mentioning that these estimates are based on the correlation of wealth across generations without controlling for earnings. As a result earnings could potentially be an omitted variable in this regression. For example, a positive shock to a grandparent's earnings can lead to an increase in wealth for the parent as well as an increase in their earnings – since earnings are persistent. Since children's wealth depend on parental earnings, it becomes correlated with parent's wealth as well. Nevertheless, Cordoba et al. (2015) show that the canonical BB model cannot generate high persistence in wealth.[2]

[2] The intergenerational persistence of wealth depends heavily on the magnitude of $\lambda'(\omega_t)$. An increase in the slope of $\lambda(\omega_t)$ can increase the persistence of wealth. In the Cordoba et al. (2015) formulation, $\lambda(\omega_t)$ is linear in $\omega_t$ and thus its slope is determined by average cost of children. The authors do not consider a modification of this functional form.

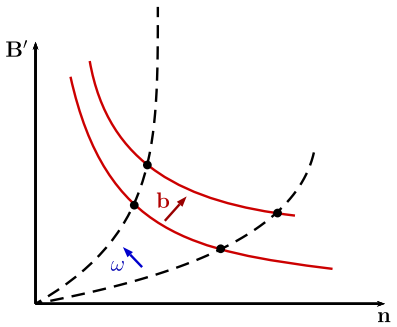


**Fig. 2.** Indifference curves and optimal choices in $(n, B')$ space in a simplified version of the model in Cordoba et al. (2015).

## 3. The mechanism in Cordoba et al. (2015)

Cordoba et al. (2015) elegantly show that the benchmark model of Barro and Becker (1989) and Alvarez (1999) can be modified to allow for a greater degree of intergenerational persistence of wealth. They allow for three modifications: (i) a non-homothetic preference towards children given by $\beta(1-e^{-\mu n})V(B'/n;\omega)$, (ii) concave cost of children, given by $\lambda(\omega)\left[(n+\kappa)^{\theta}-\kappa^{\theta}\right]$, (iii) parents can only have discrete number of children.

The first two modifications break the hometheticity of preferences between total bequests, $B'$, and family size, $n$. In particular, they both imply that wealth-expansion paths are convex as depicted in Fig. 2. Allowing for a discrete number of children also creates intergenerational persistence in per-capita wealth. In presence of this discreteness, an increase in parental wealth leads to forces towards an increase in family size. However, since increases in family size must come in large sizes, a small increase in parental wealth will lead to an increase in per capita wealth for the children.

The validity of the mechanism in Cordoba et al. (2015) relies heavily on the non-hometheticity of preferences towards children. In particular, when preferences towards children are given by $g(n)V(b';\omega)$, the expression $ng'(n)/g(n)$ can be thought of as the compensated elasticity of fertility. Cordoba et al. (2015) et al.'s preferences imply that this elasticity is decreasing in fertility. The measurement of this object in the data is particularly problematic given its definition. First, it is hard to estimate changes in the price of children – this includes opportunity cost of working time and changes in prices of goods and services related to child rearing. Second, such estimation requires large datasets containing measure of life-time income and wealth as well as data on family size.

Another possibility is to use data from relationship between wealth and fertility from both cross-section and time series. As shown by Benhabib and Nishimura (1993), in this specification, steady state wealth and fertility are negatively correlated in the aggregate. This observation is supported by long-run data. Furthermore, as shown by Cordoba et al. (2015), the wealth elasticity of fertility in their model is 0.29 while this elasticity as measured by Lovenheim and Mumford (2013), elasticity of fertility to changes in housing wealth is 0.2. This evidence is modestly consistent with the functional form assumptions made by the authors.

## 4. Alternative mechanisms

While the authors' mechanism for reconciling intergenerational persistence of wealth with altruistic fertility choice is plausible, alternative mechanisms are worth exploring. Such mechanisms can shed light on the precise nature of intergenerational transmission of wealth and earnings and can ideally be tested against the authors hypothesis. In this section, I will discuss two such mechanisms.

### 4.1. Quantity–quality trade-off

An unrealistic feature of the model discussed in Section 2 is that intergenerational transmission only occurs through monetary transfers between generations. This is despite the fact that part of intergenerational transfers are in the form of investments by parents in children's human capital. This is particularly important since most parents do not leave bequests.

With human capital considerations, parents face a quantity–quality trade-off; having a large family size comes at a cost of low investment in children's human capital and thus low future earnings. Consider a version of the problem in (1) where parents can invest in children's earnings. In particular, let $h(s_t)$ be the children's average earnings as a function of investment in human capital, $s_t$, by the parents. If shocks to earnings are uncorrelated across generations, each parent solves the following optimization problem:

$$\max_{c,B',n} \frac{c^{1-\sigma}}{1-\sigma} + \beta n^{\epsilon} V\left(B'/n, h(s)\right) \tag{2}$$

subject to

$$c+\lambda(\omega)n+B'+ns=\omega+(1+r)b$$

Equating marginal rate of substitution between $s$ and $n$ and between $B'$ and $n$ gives us the following equation:

$$\frac{\varepsilon V(B'/n,h(s))}{V_b(B'/n,h(s))}=\frac{B'}{n}+s+\lambda(\omega)$$

$$\frac{\varepsilon V(B'/n,h(s))}{V_h(B'/n,h(s))h'(s)}=\frac{B'}{n}+s+\lambda(\omega)$$

The above is a system of two equations in two unknowns and therefore $s$ and $B'$ are uniquely determined and are thus independent of parental wealth. As in the canonical BB model, intergenerational persistence occurs only through dependence of children's wealth and human capital on earnings – since $\lambda(\omega)$ is a function of parent's earning. In other words, variations in parental wealth do not independently affect parent's choice of human capital and per-capita bequests. Therefore, persistence of earnings and wealth depends on the magnitude of $\lambda'(\omega_t)$. This is a potential avenue that can be explored quantitatively.

### 4.2. Credit constraints

An important assumption in the above formulation is that parents are unconstrained in their investment in children's human capital. One potential consideration is that parents might faces credit constraints in their human capital investment. This is particularly important since as showed by Lochner and Monge-Naranjo (2011), credit constraints are an important factor that explain the rise in the importance of wealth in determining college attendance.

In the presence of credit constraints, one effect of intergenerational pecuniary transfers is that their relax credit constraints. One can thus impose the following constraint on problem (2):

$$s\leq f(b,\omega)$$

where function $f$ is increasing in both arguments. When the above constraint is binding, the cost of investing in children's human capital is low. As a result parents might decide to invest more in children via an increase in family size, i.e., along the quantity margin. An increase in $b$ relaxes this constraint and tilts the quantity–quality trade-off in favor of quality. This is depicted in Fig. 3.

Thus an increase in parental wealth leads to an increase in children's wealth and human capital. This implies that credit constraints in combination with human capital investment is another potential mechanisms that can generate intergenerational persistence in wealth and earnings.

## 5. Conclusion

Cordoba et al. (2015) is a careful study that improve our understanding of the mechanisms behind intergenerational transmission of wealth and earning in the presence of endogenous fertility. As the authors show decreasing elasticity of fertility together with concave cost of children and discrete choice of children are required to reconcile the canonical model of endogenous fertility of Barro and Becker (1989) with intergenerational persistence of wealth. As I have argued in this note, there are other mechanisms that can potentially generate intergenerational transmission of wealth and earning. A careful quantitative approach is required to disentangle the mechanisms behind intergenerational persistence which can be pursued in future work. This is especially important as various countries are struggling with aging work force and are implementing policies to revive population growth. Models such as the one developed by Cordoba et al. (2015) are crucial in understanding the effectiveness and distributional of impact of such policies.

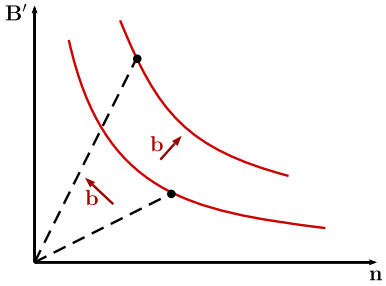


**Fig. 3.** Indifference curves and optimal choices in $(n,B')$ space with quality–quantity trade-off and credit constraints.

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