Optimal Taxation of Wealthy Individuals

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Carnegie Mellon University, March 2016

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  - Saez and Zucman (2015): share of wealth in the top 0.1%: 7% in 1978; 22% in 2013
- How should the tax schedule treat wealth/capital at the top?
- Current theories are insufficient
  - Rep. agent models: Chamley- Judd; incapable of answering this question
  - Models with labor income risk: counterfactual wealth distribution
- This paper: develop a framework to analyze optimal taxation of wealth with capital income risk

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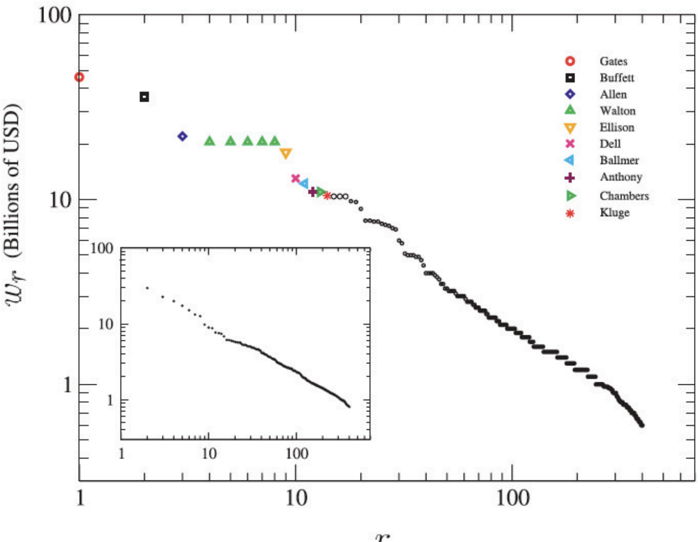
  - Models with labor income risk vs. models with capital income risk

Wealth Inequality

Power law for wealth; $\Pr(w > \hat{w}) = \kappa \hat{w}^{-\nu}$, $\nu_{data} = 2.3$



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- What does an efficient/"fair" distribution of wealth look like?

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    - Endogenous risk-return trade-off in the cross-section determines optimal taxes
    - Taxes on capital income: Progressive
        - Key intuition: progressive taxes provide insurance with respect to shocks to rate of return
    - Formula for characterizing the efficient tail of long-run wealth distribution

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  - Heterogeneity in saving motives (discount factor shocks): Saez (2002), Piketty-Saez (2012), Farhi-Werning (2013)
  - Insurance role of progressive taxes: Vickrey(1947), Friedman (1948), Varian (1980)

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- More recent literature on wealth distribution:
  - Benhabib, Bisin, and Zhu (2011), Gabaix, et. al. (2015): generate pareto distribution via rate of return shocks
    Builds on insights from Champernowne (1953) and Simon (1955)

Outline

- Two Period Model
- Infinite Horizon Model:
  - Taxes
  - Long-run distribution of wealth

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- Feasibility:

$$\begin{aligned} \int_\Theta [c_0(\theta) + k_1(\theta)]\, dF(\theta) &\leqslant e_0 \\ \int_\Theta \int_0^\infty c_1(\theta, \epsilon) dH(\epsilon) dF(\theta) &\leqslant \int_\Theta \theta k_1(\theta) dF(\theta) \end{aligned}$$

Taxation Problem

- Budget constraint:

$$c_0 + k_1 + b_1 = e_0$$
$$c_1(\epsilon) = \epsilon\theta k_1 + Rb_1 - T(\epsilon\theta k_1, Rb_1)$$

- Important assumption: No insurance against ϵ other than self insurance

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- Optimal Taxation Problem:

$$\max_{T(\cdot,\cdot)} \int_\Theta g(\theta)U(\theta)dF(\theta)$$

$$U(\theta) = \max u(c_0) + \beta \int_\epsilon u(c_1(\epsilon))dH(\epsilon)$$

subject to B.C.

Feasibility

Primal Approach

- Focus on allocations and back out taxes
- Taxation problem is equivalent to a mechanism design problem
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$$u(c_0(\theta)) + \beta \int_0^\infty u(c_1(\theta, \epsilon)) dH(\epsilon) \geqslant$$

$$u(c_0(\hat{\theta}) + k_1(\hat{\theta}) - \hat{k}) + \beta \int_0^\infty u(c_1(\hat{\theta}, \epsilon \frac{\theta \hat{k}}{\hat{\theta} k_1(\hat{\theta})})) dH(\epsilon)$$

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- choose allocations to maximize welfare subject to I.C.

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- Ex-post taxes: $T_y(\epsilon\theta k_1(\theta), Rb_1(\theta)), T_b(\epsilon\theta k_1(\theta), Rb_1(\theta))$

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- Suppose that $T(y, Rb) = (1-\tau_b)Rb + (1-\tau_k)y + T_1$
- Increase τ_k by $d\tau_k$ and T_1 by dT_1 to keep the entrepreneur indifferent:
$$\frac{k}{1-\tau_k}d\tau_k = \frac{1}{R\left(1-\tau_b\right)}dT_1$$

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  - Behavioral change

$$\tau_k \theta dk + \tau_b R db = -\left[\theta \frac{\tau_k k}{1-\tau_k}\xi^h_{k,\theta} + R\frac{\tau_b b}{1-\tau_b}\xi^h_{b,\theta}\right] d\tau_k$$

$\xi^h_{k,\theta}, \xi^h_{b,\theta}$: compensated elasticities

- Optimal Taxes: ΔMech. + ΔBeh. $= 0$

$$\theta\left(1-\tau_k\right) - R\left(1-\tau_b\right) = \theta\tau_k\xi^h_{k,\theta} + R\frac{\tau_b b(1-\tau_k)}{k(1-\tau_b)}\xi^h_{b,\theta}$$

A Preliminary Analysis

- Similar perturbation for τ_b and combine with above

$$\frac{\tau_k}{1-\tau_k} = \frac{\theta\left(1-\tau_k\right) - R\left(1-\tau_b\right)}{\xi^h_{k,\theta} - \frac{\xi^h_{k,R}}{\xi^h_{b,R}}\xi^h_{b,\theta}}$$

- Optimal taxes are a function of risk-premium and elasticities

- ξ: complicated formula; without risk:

$$\xi^h = \frac{1}{\sigma}\frac{1}{\beta + (k_1/c_0)^{1-1/\sigma}} - 1$$

- If $\sigma \geqslant 1$ decreasing with k_1/c_0.

- Increasing risk-premium and decreasing elasticities $\Rightarrow$ progressive taxes

Optimal Provision of Incentives

- Above: Form of insurance is exogenous; Now: allow optimal form of insurance

- Still assume that θ is observable; IC:

$$k_1(\theta) \in \arg\max_{\hat{k}} u(c_0(\theta) + k_1(\theta) - \hat{k}) + \beta \int_0^\infty u(c_1(\theta, \frac{\hat{k}}{k_1(\theta)}\epsilon)) dH(\epsilon)$$

- Local version:

$$u'(c_0(\theta)) = \beta \int_{\mathbb{R}_+} u(c_1(\theta, \epsilon)) \frac{\rho(\epsilon)}{k_1(\theta)} dH(\epsilon)$$

Incentive Compatbility

Assumption

1. *The distribution of* ε *satisfies the following:*
   1.1 *The support of* H *is the set of all positive real numbers,* $\mathbb{R}_+$*,*
   1.2 *The distribution function* H *satisfies monotone likehood ratio assumption, i.e.,* $\rho(\varepsilon) = -\frac{\varepsilon h'(\varepsilon)}{h(\varepsilon)} - 1$ *is increasing in* ε*.*
   1.3 *The monotone likelihood ratio takes a finite value* $\varepsilon = 0$*, i.e.,* $\lim_{\varepsilon \to 0} -\frac{\varepsilon h'(\varepsilon)}{h(\varepsilon)} = \rho(0) + 1 > -\infty$.
2. *Preferences satisfy:* $\sigma \geqslant 1/2$

- Guarantees local IC implies IC
- Examples for H: generalized Gamma distribution (Weibul, Gamma, Exponential, etc.); Log-normal does not work

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- Optimal consumption schedule at $t = 1$:

Proposition

1. *Suppose that* $\theta \leqslant \bar{\theta}$. *Then, any optimal allocation,* $c_1(\theta, \epsilon)$ *satisfies*

$$\frac{1}{u'(c_1(\theta, \epsilon))} = \alpha g(\theta) \left[1 + s(\theta)\rho(\epsilon)\right]$$

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2. *Sensitivity,* $s(\theta)$*, is strictly increasing in* θ

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- Loosely speaking: Endogenously determined risk-return trade-off

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- Progressivity is a natural result: it provide insurance.
- Next: An Inverse Euler Equation to understand this better
- A perturbation

$$\begin{aligned}\Delta c_0(\theta) &= \frac{\delta}{u'(c_0)}\\ \Delta c_1(\theta) &= -\frac{\beta\delta}{u'(c_1)}\\ \Delta k(\theta) &= \sigma\frac{1}{u'(c_0)}\frac{k}{c_0}\delta\end{aligned}$$

  - keeps individual utility unchanged
  - Δk: similar to compensated elasticity

Modified Inverse Euler Equation

- Optimality implies that cost of this perturbation must be zero:

$$\sigma\frac{\theta - R}{R}\frac{k_1}{c_0} = 1 - \frac{u'(c_0)}{\beta R E\left[u'(c_1)\right]}$$

- LHS is increasing in θ. RHS is measure of distortions

Proposition

Optimal wedges $\tau_k(\theta), \tau_b(\theta)$ *are increasing in* θ*.*

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- Inside saving/equity:
  - Two opposing effects: higher taxes give more insurance; less incentive to invest
  - If sensitivity is mildly increasing, then insurance effect dominates
  - In the paper: this is the case! (had to resort to Chebyshev's integral inequality!)

Private Productivities

- Analyze using the first order approach:

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\begin{aligned}
u'(c_0(\theta)) &= \beta \int_{\mathbb{R}_+} u(c_1(\theta, \epsilon)) \frac{\rho(\epsilon)}{k_1(\theta)} dH(\epsilon) \\
U'(\theta) &= \frac{1}{\theta} k_1(\theta) u'(c_0(\theta))
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- The same Inver Euler Equation holds, therefore

Proposition

Suppose that $k_1(\theta)/c_0(\theta)$ *is increasing, then* $\tau_b(\theta)$ *is increasing with private* θ.

Private Productivities

- No distortions at the limits:

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Proposition

At the optimum, $\tau_b^{PI}(\theta; R) = \tau_b^{FI}(\theta; R)$, $\tau_k^{PI}(\theta; R) = \tau_k^{F}I(\theta; R)$, *when* $\theta = R, \bar{\theta}$

Numerical Exercise

- A period is around 25 years;
  - $\beta = (0.95)^{25}$
  - $R = 1/\beta \approx 2.77$

- preferences: $\sigma = 1$

- Not a lot of evidence on ϵ and θ.
  - Campbell et. al. (2001): st. dev. publicly traded stocks: 50%
  - DeBacker, Panousi and Ramnath (2015): Tax data on business income
    - Annual st. dev is around 46%
    - 23% of variation from individual heterogeneity; 77% from risk

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- $\epsilon \sim \Gamma$: $var[\epsilon] \in \left\{(0.2)^2 \times 25, (0.36)^2 \times 25, (0.45)^2 \times 25\right\}$

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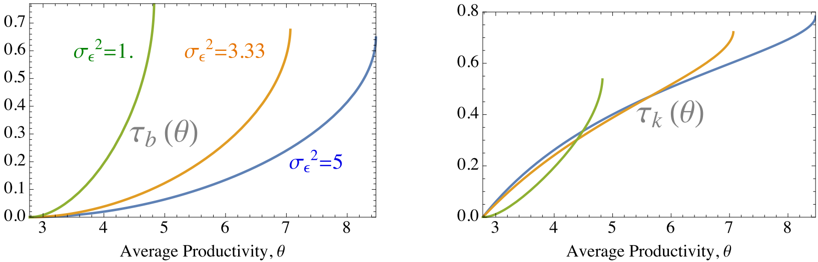
- $\theta \in [R, \theta_{max}], f(\theta) \propto 1/\theta$
  $\theta_{max} \in \left\{(1 + 0.065)^{25}, (1 + 0.083)^{25}, (1 + 0.09)^{25}\right\}$

Optimal Taxes



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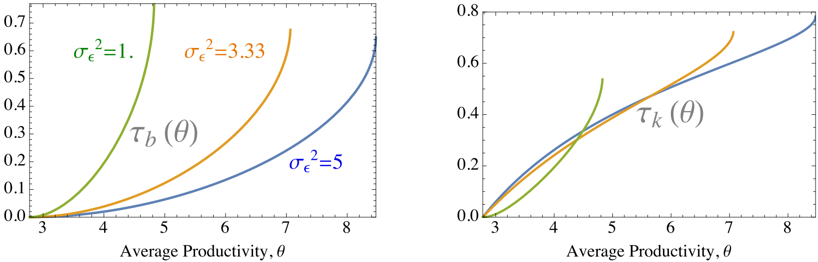
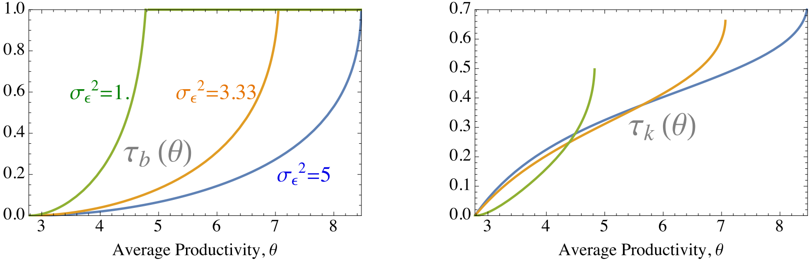


- Private information does not make much of a difference: $\Delta\tau \approx 10^{-3}$

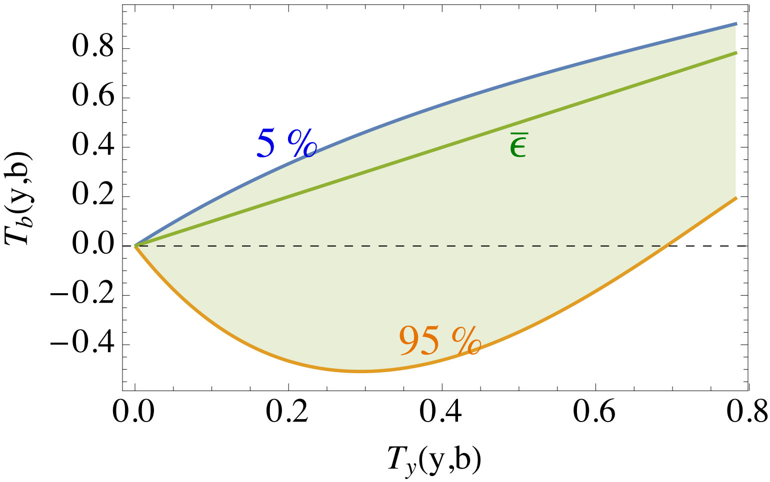
Equivalent Income Taxes

- Income Tax:

$$\left(1 + r(1 - \hat{\tau})\right)^{25} = R(1 - \tau)$$



Ex-post Taxes



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  - (Efficient) Long-run distribution of wealth

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  - Altruistic toward future generations

Dynamic Extension

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  - Draw $\theta_t \sim F(\theta_t)$; i.i.d. across generations and households.

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- Provide labor to the old running the firm

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- Information:
  - $k_{t+1}, c_{0,t}, c_{1,t}, \epsilon_{t+1}, \theta_t$ are private
  - y_{t+1}, l_{t+1} as well as bequests are observable.

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  - Separate each generations problem from each other – component planning problem
  - Focus on local incentive constraints

Recursive Problem

Component planning problem in a stationary economy:

$$
\begin{aligned}
P(w) &= \max_{c_0, c_1, k, w', U} \int_{\Theta} \left[\alpha q \kappa \theta k(\theta) - c_0(\theta) - k(\theta) + p \right. \\
&\quad \left. + q \int_0^{\infty} \left[-c_1(\theta, y) + qP\left(w'(\theta, y)\right)\right] dG\left(y | \theta, k(\theta)\right) dF
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- Qualitative result: progressive taxes on (outside) saving; similar for inside saving

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- Compare with models with labor income risk

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- Log-preferences: $q^2 = \hat{\beta}$

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- Wealth converges to zero almost surely; households borrow against the labor/capital income of their children

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- Implication: $(1+r)^{-2} = q^2 > \hat{\beta}$

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Tail: $\Pr\left[P(w_t) > A\right] \propto A^{-\nu}$

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Efficient Tail:

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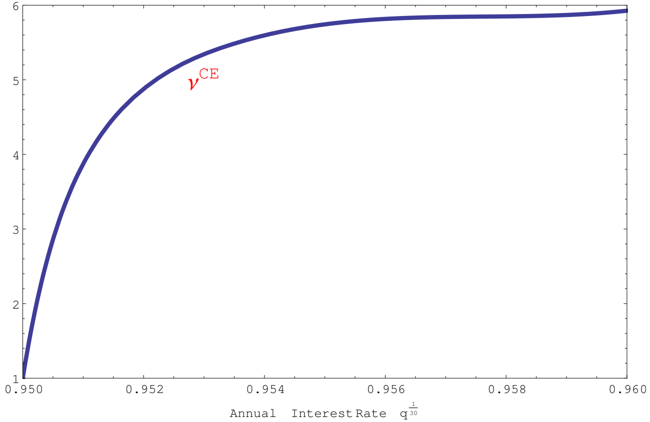

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Incomplete Market:

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q	ν^{IM}
$0.95^{30} - 0.96^{30}$	Non–Stationary
1.0309^{30}	1
1.0319^{30}	1.0423
1.0329^{30}	1.0872

The tail behavior of stationary distribution in the incomplete market model

Conclusion

- Developed a theory of optimal capital taxes consistent with wealth distribution
- Method for characterization of efficient wealth distribution
- Possible application: human capital accumulation and distribution of income