

The Endogenous Grid Method

Here I quickly describe how to use the endogenous grid method to solve the income fluctuations problem.¹ Note that the policy function $a'(a, y)$ must satisfy the following Euler equation:

$$u'(Ra + y - a'(a, y)) \geq \beta R \sum_{y'} \pi(y'|y) u'(Ra'(a, y) + y' - a'(a'(a, y), y'))$$

with equality if $a'(a, y) > \underline{a}$.

In order to find a policy function that satisfies the above, proceed as follows:

1. Specify a grid $A = \{a_1, a_2, \dots, a_M\}$ with $a_1 = \underline{a}$. This is a fixed grid that will not change throughout the algorithm.
2. Guess an initial vector of values for the policy function on the grid \hat{a}_{ij} . Use a linear interpolation and called the resulting function $\hat{a}(a, y)$.
3. For each point on the grid (a_i, y_j) , consider the following equation in a^* :

$$u'(Ra_{ij}^* + y_j - a_i) = \beta R \sum_{y'} u'(Ra_i + y' - \hat{a}(a_i, \hat{y}))$$

This equation can be easily solved since its solution is given by

$$a_{ij}^* = \frac{(u')^{-1}(\beta R \sum_{y'} u'(Ra_i + y' - \hat{a}(a_i, \hat{y}))) + a_i - y_j}{R}$$

Note that a_{ij}^* represent the initial value of asset that together with income y_j leads to asset level a_i tomorrow; given the fact that we will use the policy function $\hat{a}(\cdot, \cdot)$ from tomorrow onwards. At the end of this step, we have a vector of current assets as a function of current income and future asset. That is, this function maps a_{ij}^* and y_j into a_i .

4. Use a_{ij}^*, y_j as your new grid and use a linear interpolation to form a function $\tilde{a}(a, y)$. Using this linear interpolation, we can calculate $\tilde{a}_{ij} = \tilde{a}(a_i, y_j)$. If for some values of a_i this falls below \underline{a} , replace it with \underline{a} .
5. Now compare \tilde{a} with \hat{a} . If the two are close according to some criterion, say the maximum difference over the grid is below a thoreshold, we are done. If not, replace \hat{a} with \tilde{a} and repeat steps 2 through 4.

¹This note is based on Fabrizio Perri's notes on this method.