47802 - Macroeconomics 1

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Final Exam

Problem 1. A Model of On-the-Job Search Consider the following version of the McCall search model that allows for on-the-job-search. In particular, suppose that time is continuous and workers - employed or unemployed discount the future at rate ρ. A worker that has a job separates from her job at rate η. An unemployed worker finds a job at rate λ_0 - the wage is then drawn from a distribution $F(w)$ which is continuous. Moreover, a worker who has a job gets an opportunity to draw a new wage offer at rate λ_1 - again from the same distribution $F(w)$. The worker then decides whether to accept this new offer or not.

As usual, assume that the flow value of unemployment is b; the workers are risk-neutral and cannot save.

a. Let $V(w)$ be the value of a job at hand that pays the wage w . Let U be the value of unemployment. Write the Bellman equation for U and $V(w)$.

b. Calculate the reservation wage, w^*, for an unemployed worker as a function of the value function $V(w)$. Under what conditions, $w^* < b$ holds? Provide an intuition for this result.

c. What is the reservation wage for an employed worker?

d. How does an increase in unemployment benefits, b, affect the job-to-job turnovers? Show your result and explain intuitively.

Problem 2. An Stochastic AK economy

Consider an economy in which production is done only with capital and output is given by

$$Y_t = A_t K_t$$

where A_t is an i.i.d. stochastic process and is distributed according to the c.d.f. $F(A)$. Capital depreciates at rate δ. Suppose that the economy exhibits a representative agent and that preferences of such hypothetical agent is given by

$$\sum_{t=0}^{\infty} \beta^t u(C_t)$$

where $u(\cdot)$ is a CRRA utility function given by

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & \gamma \neq 1 \\ \log c & \gamma = 1 \end{cases}$$

a. Define a Competitive Equilibrium for this economy assuming that trading occurs in sequential asset markets.

b. State the First Welfare Theorem and use it to formulate a competitive equilibrium as a solution to a planning problem.

c. Formulate the planning problem problem recursively. You are required to use only one state variable.

d. Solve the functional equation associated with this recursive problem and find the value function and policy functions using a guess and verify method. What assumptions on these fundamentals should be made so that the solution to this problem exists and is unique?

e. Using the policy functions calculated above, calculate the average growth rate of this economy - assuming that the data comes from the stationary distribution of the model. The result should be expressed in terms of the fundamentals parameters of the model as well as various moments of the distribution $F(\cdot)$.

In what follows, assume that $\delta = 1$ and that

$$F(A) = \int_0^A \frac{1}{\sigma x \sqrt{2\pi}} e^{-\frac{\left(\log x + \sigma^2/2 - \log \overline{A}\right)^2}{2\sigma^2}} dx$$

That is, A is distributed according to a log-normal distribution whose mean is $\overline{A}$ and its variance is $\left(e^{\sigma^2} - 1\right) \overline{A}^2$.

f. What is the average growth rate of the economy? How does growth interact with risk, i.e., how does the average growth rate of the economy depend on σ? Explain your answer intuitively.

g. Calculate the price of a risk-free real bond, P_t^f. What is the risk-free rate in this economy.

h. Calculate the price, P_t, of holding a firm that owns the physical capital and invests from the output that it produces and show that $P_t = K_t$; Hint: This is an asset with a dividend process given by $D_t = A_t K_t - K_{t+1}$. What is the return on such an asset?

i. Calculate the equity premium as the difference between average return on physical capital and the risk-free rate. How does this depend on risk-aversion, γ, and variance of the shocks, σ. Explain intuitively.