McCall's Model of Sequential Search

So far, in our models, we have discussed employment but not unemployment. This is mainly because of the intricate definition of unemployment. It is official defined as the fraction of the labor force that does not have a job where labor force is defined as people who either have a job or are actively looking for one. One way to think about unemployment is as stemming from search frictions: workers cannot necessarily find all the jobs and they might find a job that they do not like.

The McCall search model is a simple partial equilibrium model that captures this idea. Suppose that the economy is populated by a continuum of workers and a continuum of jobs. Jobs have wages attached to them and are assumed to be heterogeneous in the wage they offer. Suppose that wages are distributed according to a c.d.f. function $F(w)$ with $Supp(F) = [\underline{w}, \overline{w}]$ and that if a job gets taken by a worker, there will be another with the same wage and therefore, unemployed workers always face the same distribution of wage offers. We assume that once workers accept a job offer, they will work at this job forever. We assume that workers are risk neutral; cannot save and discount the future at rate β. Moreover, they find a job with probability λ. Also, we assume that unemployed workers receive a benefit b when they are unemployed; this could be a stand-in for unemployment benefits, value of home production, leisure, etc.

The easiest way to analyze this problem is by working directly with value functions. Let $V(w)$ be value of being employed at a job that pays w and let U be the value of being unemployed. Then

$$U = b + \beta\lambda \int \max\{V(w), U\}\, dF(w) + \beta(1-\lambda)U$$
$$V(w) = \frac{w}{1-\beta}$$

The first equation captures the idea that value of unemployment is equal to its flow value plus a continuation value. When an unemployed worker finds a job - with probability λ - she can decide whether to accept this job and get a value of $V(w)$ or to reject and get value associated with unemployment, U. With a complimentary probability, she will not find a job in which case her value is simply U. The second equation simply states that the value of employment at a job with wage w is $\frac{w}{1-\beta}$ the value of earning w forever. We can write the first equation as

$$U - \beta U = b + \beta\lambda \int \max\left\{\frac{w}{1-\beta} - U, 0\right\} dF(w)$$

Thus a reservation wage w^* exists for which the unemployed accept all offers above w^* and reject all offer below. Furthermore, we must have $U = \frac{w^*}{1-\beta}$. We have

$$w^* = b + \beta\lambda \int_{w^*}^{\overline{w}} \frac{w - w^*}{1-\beta} dF$$
$$w^* = b + \frac{\beta}{1-\beta}\lambda \int_{w^*}^{\overline{w}} (w - w^*)\, dF$$

This equation is intuitive. It states that the reservation wage is equal to unemployment benefit plus the option value of waiting for a new offer as captured by the second term in the RHS. The RHS is decreasing in w^*, while the LHS is increasing. Therefore, a unique solution to above exists. If we assume that $\overline{w} > b$, then the solution to the above is higher than b and lower than $\overline{w}$. In addition, it can be shown that an increase in b leads to an increase in w^*. To see this, we have

$$dw^* = db + \frac{\beta}{1-\beta}\lambda \int_{w^*}^{\overline{w}} (-dw^*)\, dF$$
$$\left[1 + \frac{\beta\lambda}{1-\beta}\left(1 - F\left(w^*\right)\right)\right] dw^* = db$$
$$\frac{dw^*}{db} = \frac{1}{1 + \frac{\beta\lambda}{1-\beta}\left(1 - F\left(w^*\right)\right)}$$

So this model captures the idea that an increase in unemployment benefits makes the workers more picky with respect to jobs and leads to higher unemployment.

The McCall search model is very elegant and simple yet it is potentially rich enough to consider various extensions and modifications. For example, one can easily think about exogenous separation in this model. That is suppose that jobs terminate with probability η in each period. Then, the value of unemployed is the same as before. However, the value of being employed is not given by

$$V(w) = w + \beta(1-\eta)V(w) + \beta\eta U \to V(w) = \frac{w + \beta\eta U}{1 - \beta(1-\eta)}$$

so now the two values are interdependent. The simplicity of the model also allows us to examine various moments. For example, we can think about measure of wage dispersion in this model - since wage dispersion is endogenous. One statistic related to that is the min-mean ratio: the ratio of minimum observed wage to average wage

$$\frac{w^*}{w_m} = \frac{w^*}{\mathbb{E}\left[w|w \geq w^*\right]} = \frac{b + \frac{\beta}{1-\beta}\lambda\left[1 - F\left(w^*\right)\right]\left(w_m - w^*\right)}{w_m}$$

$$\frac{w^*}{w_m} = \frac{\frac{b}{w_m} + \frac{\beta}{1-\beta}\lambda\left[1 - F\left(w^*\right)\right]}{1 + \frac{\beta}{1-\beta}\lambda\left[1 - F\left(w^*\right)\right]}$$

In the above formula, $\lambda\left(1 - F\left(w^*\right)\right)$ is the rate at which the unemployed find jobs and thus can be measured directly in the data. A paper by Hornstein et al. (2011) does the measurement on the RHS and shows that the model cannot explain the min-mean ratio observed in the data.

Sometimes, in search models, it is useful to formulate the problem in continuous time. This is because some Bellman equations are easier to analyze in continuous time. To do this, I consider the above model and will try to send the time interval to zero. In particular, suppose that the length of the time interval is Δ. Furthermore, suppose that the probability of finding a job is given by $\eta\Delta$ and the discount factor is given by

$$\frac{1}{1 + \rho\Delta}$$

Note that as we send Δ to zero, the discount factor converges to 1 and the probability of finding a job converges to 0. This makes sense as the probability of finding a job in the next instant is probably 0. With continuous time, the appropriate way to think about η and ρ is rate at which the probability of finding a job increases with time and the rate at which the individual discounts the future, respectively. Note also that as we send Δ to zero, we should also decrease wages and unemployment benefit. As in the case of job finding rate, we can talk about wage rate and benefit rate. In other words, for any period of length Δ, $b\Delta$ is the unemployment benefit received by the individual during the period and $w\Delta$ is the wage received by a worker. Therefore, we have

$$V_\Delta\left(w\right) = \frac{w\Delta}{1 - \frac{1}{1+\rho\Delta}} = \frac{w\left(1 + \rho\Delta\right)}{\rho}$$

$$\left(1 - \frac{1}{1 + \rho\Delta}\right)U_\Delta = b\Delta + \frac{\eta\Delta}{1 + \rho\Delta}\int \max\left\{V_\Delta\left(w\right) - U_\Delta, 0\right\}dF\left(w\right)$$

$$\frac{\rho}{1 + \rho\Delta}U_\Delta = b + \frac{\eta}{1 + \rho\Delta}\int \max\left\{\frac{w\left(1 + \rho\Delta\right)}{\rho} - U_\Delta, 0\right\}dF\left(w\right)$$

Therefore, as Δ converges to 0, the limit of the above are given by

$$V(w) = \frac{w}{\rho}$$
$$\rho U = b + \eta \int \max\left\{\frac{w}{\rho} - U, 0\right\} dF(w)$$

The above value functions can be explained in words. The LHS is the decline in the continuation value of unemployed from one period to the next (infinitesimally next!):

$$\frac{d}{dt}\left(U - e^{-\rho t} U\right)\Big|_{t=0} = \rho U$$

The right hand side is the increase in the value of being unemployed. The unemployed person, collects benefits for sure and at rate η will find a job that she accepts if its value is above U. If w^* is the reservation wage, then $U = \frac{w^*}{\rho}$ and

$$w^* = b + \frac{\eta}{\rho} \int_{w^*}^{\overline{w}} (w - w^*)\, dF(w)$$

We can write the above as

$$\rho w^* = \rho b + \eta \int_{w^*}^{\overline{w}} (w - w^*)\, dF(w)$$

If we send ρ to 0, the above implies that w^* converges to $\overline{w}$. In other words, if the individual does not discount the future, then she becomes extremely picky and will only accept the best wage offer out there.

It is also easy to calculate various moments related to unemployment in this model. For example, the rate at which an unemployed person finds a job is given by

$$H = \eta\left[1 - F(w^*)\right]$$

To see this note that unemployment must satisfy

$$\dot{u}_t = -\eta\left[1 - F(w^*)\right] u_t$$

because in each period, an unemployed person finds a job at rate η and will accept it with probability $1 - F(w^*)$. That is the probability of a person who is unemployed at time 0 has found a job by t is $1 - e^{-Ht}$ whose density is given by He^{-Ht}. Therefore, average unemployment duration is given by

$$\int_0^\infty tHe^{-Ht} dt = -\int_0^\infty t d\left(e^{-Ht}\right) = \int_0^\infty e^{-Ht} dt = \frac{1}{H}$$

Thus average duration of unemployment is $\frac{1}{\eta[1-F(w^*)]}$. This would have been harder to calculate in discrete time.

The Rothschild Critique and Diamond Paradox

In the above example, an important assumption is that wages are exogenously given. The question is can we get this to come out of optimal behavior by firms. As it is suggested by the title, the result is negative. In fact, Rothschild critique and Dimaond paradox illustrate that it is difficult to get a wage distribution to come out of optimizing behavior by firms. To see this, suppose that firms are heterogeneous with respect to their productivity, z; they are capacity-constrained in that they only have one job and post their wage at time 0 and commit to this posted wage. Taking as given the reservation wage policy of the individual, it is clear that no firm should offer something higher than w^*. To see this suppose that some firm is posting a wage above the reservation wage w^*, then this firm can cut its wage by $\varepsilon > 0$ and small; workers will still accept the job offer since this lower wage is still above the reservation wage and thus the profits for the firm will go up. In other words, there wont be an equilibrium distribution of wages. Diamond (1971) went one step further. He said, suppose that all firms are posting a wage $w^* > b$. Then as long as $\beta < 1$, one firm has an incentive to deviate and post something lower than w^*. To see this, suppose that one firm posts a price $w^* - \varepsilon$. The value of accepting this offer for an unemployed worker is

$$\frac{w^* - \varepsilon}{1 - \beta}$$

while if she waits, her payoff is, at best, given by

$$U = b + \beta\eta \frac{w^*}{1 - \beta} + \beta\left(1 - \eta\right) U$$

which is less than $\frac{w^*}{1-\beta}$. Therefor, for $\varepsilon > 0$ and small enough, the value of accepting the offer $w^* - \varepsilon$ is higher than the value of waiting. This implies that in equilibrium, $w^* = b$. In other words, firms will fully exploit the workers and offer them the value of unemployment benefit and workers will accept any job offer right away. In other words, they are always indifferent between working and not working. This is puzzling because presumably when a firm is setting a wage today, they should be thinking that the worker always has the option to wait. But the existence of search cost makes a firm who has met the worker effectively a monopolist.

We will talk about two models that will resolve this issue. First, a model where some workers have two offers in their hands - thereby creating some sort of competition within a period among firms. Second, a model where firms do not commit to wages - no wage posting - and wages are determined as a result of a bargaining process.

The Burdett-Judd Model

The first idea was explored by Burdett and Judd (1983). To see a very simple version, suppose that now the economy is static. There are two firms that are identical and they post wages. There is one worker! The firms send out fliers and the worker might receive these fliers. The probability that the worker receives the flier of each firm is given by π. Therefore, there will be four possibilities for the worker: receive two fliers, receive a flier from firm 1 (or 2) and receive no fliers. The rest is the same as before: value of unemployment is b. Additionally, value of firm productivity is z and $z > b$ so it is always efficient for the worker to work for the firm.

Suppose, for now, that the firm can tell if a worker has both fliers or only one. Then the outcome is clear: when the worker is in contact with one firm, the firm basically offers her b and she will accept - basically the firm is a monopolist in this case. If the worker has two offers, then the firm knows that it is in competition with another firm and this competition leads to a wage equal to z. Again, we wont have a non-trivial distribution of wages.

Now, suppose that firms cannot tell whether a worker that they has their flier has received a flier from the other firm or not. Then they have to offer a single wage. This wage cannot be equal to their productivity, z, this is because they can always guarantee for themselves an expected profit of $(1-\pi)(z-b)$ - by basically offering b and only having a captive worker work for them. At the same time, in any symmetric equilibrium, if a firm offers any wage below z, its profits are given by

$$\left(\pi(1-\pi)+\frac{\pi^2}{2}\right)(z-w) \tag{1}$$

That is, it hires a non-captive worker with probability 1/2 and hires a captive worker for sure. Given this, the firm has an incentive to increase the wage by ε and trading with a captive worker for sure. Its expected profits are given by

$$z-w-\varepsilon$$

For $\varepsilon > 0$ and small enough, the above value is higher than (1). This means that this game between the firms has no pure strategy symmetric equilibria. It, therefore, must have a mixed strategy equilibrium. Let the c.d.f. of the wage distribution used by each firm be given by $F(w)$. Then the payoff for a firm of offering any wage w is given by

$$\pi\left(1-\pi+\pi F\left(w\right)\right)\left(z-w\right)$$

if $F(\cdot)$ does not have a mass point at w. That is, the firm will always hires a captive worker but is only able to hire a non-captive worker with probability $F(w)$ which is the probability that the other offer at hand is below w . Note that, if $F(w)$ has a mass point at w, i.e., a discontinuity, the payoff of the firm is

$$\pi\left(1-\pi+\pi F^{-}\left(w\right)+\frac{\pi}{2}\left(F\left(w\right)-F^{-}\left(w\right)\right)\right)\left(z-w\right)$$

where $F^{-}(w)$ is the left limit of F at w. The above states that if $F(w)$ has a mass point at w, then a firm offering w will get the non-captive worker who has an offer at hand of w with probability $\frac{1}{2}$.

It can be shown that $F(w)$ is nice looking! That is it has no holes and no mass point. To see this, suppose that $F(w)$ has a mass point at w which means the firm get a non-captive worker that has an offer w at hand with probability $1/2$. Now an increase in wages by ε causes the firm to attract the non-captive worker with probability w for sure and some more people. In other words, the probability of hiring a worker jumps upward but the payoff per worker declines by a small amount. This means that this will be a profitable deviation. Now, suppose that $F(w)$ has a hole in the interval $[w_1, w_2]$ with $F(w_1) = F(w_2) < 1$, then an offer of $w_2 - \varepsilon$ attracts the non-captive worker with the same probability but the profits per worker go up and thus it is a profitable deviation.

So we have shown that the distribution of wages, $F(w)$, is well-behaved and has full support over an interval $[w_1, w_2]$. Suppose that $w_1 > b$, then a deviation to b increases profits since at w_1 the probability of trading with the non-captive worker is 0 and thus lowering the wage only increases the profits from the captive worker. This implies that the profits associated with $w = b$ is given by

$$\left(1-\pi\right)\left(z-b\right).$$

Therefore, as in any mixed strategy equilibrium, any wage offered in equilibrium must satisfy

$$\left(1-\pi+\pi F\left(w\right)\right)\left(z-w\right)=\left(1-\pi\right)\left(z-b\right)$$

This pins down the distribution of wages:

$$F\left(w\right)=\frac{\left(1-\pi\right)\left(w-b\right)}{\pi\left(z-w\right)}$$

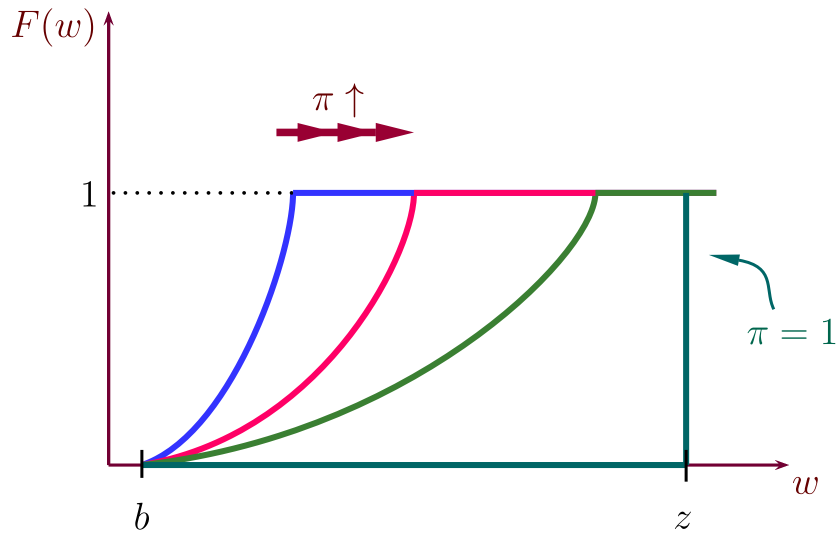

Figure 1: Changes in the distribution of wages in response to change in π

The upperbound of the distribution is given by

$$1 = \frac{(1-\pi)(w_2 - b)}{\pi(z - w_2)} \rightarrow w_2 = \pi z + (1-\pi) b$$

As π converges to 1, the upper bound converges to z and the distribution converges to 0 everywhere except at $w = z$. In other words, the distribution converges to a Dirac delta at $w = z$. The following picture depicts what happens to $F(w)$ as π increases.

Another approach that is very commonly now used in macro is to use Nash bargaining for determination of wages as we show next.

References

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