47802 - Macroeconomics I

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Midterm Exam

You have 110 minutes

Solve one of problem 1 and 2 as well as problem 3.

1. An Economy with Heterogeneity

Consider an infinite-horizon economy with a single consumption good, c, in every period and leisure, ℓ.

The economy is comprised of 2 consumers. Each has preference of the following form:

$$\sum_{t=0}^{\infty} \beta^t [u(c_{t,i}) + \gamma_i v(\ell_{t,i})], \quad i = 1, 2$$

where u and v are continuously differentiable, strictly increasing and strictly concave functions and $\gamma_1 > \gamma_2 > 0$. Suppose also that $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $v(\ell) = \psi \frac{\ell^{1-\sigma}}{1-\sigma}$ for $\sigma \neq 1$. When $\sigma = 1$, $u(c) = \log c$ and $v(\ell) = \psi \log \ell$.

Each agent in every period is endowed with 1 unit of leisure and 0 units of consumption good. There is a single firm with the following production technology:

$$Y = \left\{(n, y); (n, y) \in \mathbb{R}_+^2, y \leq An\right\}$$

for some $A > 0$.

a. (5 points) Define an Arrow-Debreu equilibrium for this economy.

b. (5 points) Assume an interior equilibrium. Calculate the prices explicitly.

c. (10 points) Show that $c_{t,1} < c_{t,2}$ and $\ell_{t,1} > \ell_{t,2}$.

d. (10 points) Now suppose that $\sigma = 1$. Set up and solve the social planner's problem. Find a condition on welfare weights so that the result in part c still holds.

2. Dynamic Programming

Consider the following sequence problem:

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

s.t.

$$\begin{aligned} c_t + k_{t+1} &\leq F(k_t) \\ k_{t+1} &\geq 0 \;\; \forall t \geq 0, \\ k_0 &: \text{ given} \end{aligned}$$

a. (5 points) Write this problem in canonical form.

b. (15 points) Assume $u(c) = -e^{-\alpha c}$ and $F(k) = Ak$ where $A > 1$. Write the Bellman equation and solve for the value function. Hint: Guess that $v(k) = -Ce^{-\gamma k}$ for some value of C and γ and verify. Ignore the positive consumption constraint.

c. (10 points) Assume $u(\cdot)$ and $F(\cdot)$ are strictly concave and continuously differentiable. Assume that the solution to the RHS of Bellman equation is interior. Show that policy function is increasing in capital. You may use theorems from Stockey-Lucas-Prescott as long as you state them and show that it is valid to use them. Moreover, assume that the solution of the Bellman equation exists and is bounded and continuous.

3. Optimal Taxation with Endogenous Government Spending

Consider the baseline one-sector growth model but assume that in each period there is an additional consumption good: a public good that is provided by the government. Suppose that utility of the representative household is given by

$$u(c, g, \ell) = \frac{c^{1-\sigma}}{1-\sigma} + \psi \frac{g^{1-\sigma}}{1-\sigma} + \zeta \frac{\ell^{1-\sigma}}{1-\sigma}$$

when $\sigma \neq 1$ and the utility for $\sigma = 1$ is defined as usual - see problem 1.

Assume that the relative price of the public good to private consumption is 1. Production of the public and private goods are done in firms using a Cobb-Douglas production function which is the same for both goods.

Households are standard. They own the capital stock and are endowed with leisure. They provide labor and rent out capital to firms and they consume the two consumption good. Note that households do not purchase the public good - this is determined by the government.

a. (5 points) Define a TDCE for this economy assuming that the government can impose linear taxes on households - their various sources of income and consumption of the consumption good as well as investment in physical capital.

b. (5 points) Derivate the implementability condition for this economy and show that an allocation is part of a TDCE if and only if it is feasible and satisfies the implementability condition.

c. (5 points) Formulate the Ramsey problem.

d. (10 points) Calculate the long-ratio of optimal government spending to GDP.

e. (10 points) What is the long-run value of optimal capital income taxes? What is the long-run value of optimal labor income taxes?