Problem Set 1

Macro 2 - Spring 2020

Problem 1. Technology shocks in the DMP Model

Consider the DMP model in Steady State and consider an infinitesimal change in productivity, z.

1. Calculate the change in wages, unemployment rate and market tightness in the Steady State.

2. Now, you should try to calibrate the model and get a sense of how whether the results match the data. Describe what information you need to calibrate this model – you can do this calibration assuming that productivity takes its historical average value. Find the appropriate data; CPS (provided by BLS) contains the data on job flows and unemployment rate. JOLTS (Job Openings and Labor Turnover Survey) contains data on vacancies. BEA contains data on GDP.
   More specifically list the set of parameters of the model that you should find their values. Describe what moments in the data pin down each parameter and find their values.
   Note: For simplicity, you can assume that matching function takes the form
   $$M\left(u,v\right)=Au^{\alpha}v^{1-\alpha}$$

3. Using the calibrated model above consider a one percent temporary increase in productivity. What is the change in the unmployment rate? As a very very rough comparison to the data, what is the ratio of standard deviation of unemployment to that of detrended log labor productivity (GDP per worker detrended using HP filter)? How does this compare to the number you found from the model?

4. The above calculation is very very rough! In this part, we will allow for shocks in the DMP model to better answer it. In particular, suppose that the labor productivity follows a stationary stochastic process as follows: at every moment, a poisson shock arrives at rate ζ and then labor productivity is drawn from a distribution $H\left(z\right)$. In case the poisson shock does not arrive, productivity remains at its current level. Define a recursive competitive equilibrium for this economy assuming that the economy is stationary, i.e., has settled to its stationary distribution. Write the set of equations – value functions, unemployment, wages, etc. – that describe the equilibrium of this economy.

5. Use the data on labor productivity (GDP per worker) in the U.S. and apply an HP-filter to it. Given this series, estimate the parameters governing the stochastic process described in part 3. You may assume that $H\left(z\right)$ has a pareto distribution. Using this estimated distribution and the parameters calibrated in part 2, solve for the equilibrium behavior of unemployment, wages and vacancies. How do these compare with the data?

Problem 2. McCall meets DMP

Consider the standard DMP model discussed in class with the following modification: when a worker and a firm meet they draw a match specific productivity, z from a continuous distribution $G\left(z\right)$. Suppose that z has a pareto distribution given by

$$G\left(z\right)=1-\left(\frac{z}{z_0}\right)^{-\gamma}$$

This productivity lasts for the duration of the job - until separation. The rest is identical to the model in class - separation rate is independent of the job productivity. Suppose that for each level of productivity that is accepted by firms and workers, there is an equilibrium wage $w(z)$. Assume that this wage is determined via Nash bargaining between the firm and the worker.

1. Write the Bellman equations for the workers and firms as a function of the wage function $w(z)$.
2. Characterize the value of reservation wage.
3. Derive the steady-state equilibrium conditions.
4. How does the wage distribution depend on unemployment benefits?

Problem 3. Burdett and Judd with heterogeneous firms

Consider the following version of the Burdett and Judd model: The model is static. Suppose that a unit continuum of firms can produce a good at constant marginal cost, c where c is heterogeneous across firms and distributed uniformly over the interval $[\underline{c}, \overline{c}]$. We refer to the distribution of marginal cost as $F(c)$; $F(\cdot)$ is a c.d.f. Suppose that there is a unit continuum of consumers who want to purchase one unit of the good. The value of the good for them is u while if they pay a price p for the good, their utility is given by $u - p$.

Consumers search for firms that have the product. A consumer meets one firm with probability q_1 while she meets two firms with probability q_2 where $q_1 + q_2 \leq 1$. Let us assume that $q_0 = 1 - q_1 - q_2$ is the probability that a consumer meets no firm. The value of a consumer who does not meet a firm is normalized to 0. A consumer with two price offers chooses the one with the lower price. Let us assume that a firm with mraginal cost c sets a price of $p(c)$.

As in Burdett and Judd, firms do not observe the number of other firms a consumer has met.

1. Supose that $p(c)$ is a strictly increasing and continuously differentiable in c. Moreover, suppose that distribution of prices is given by $G(p)$. How are $G(p)$, $F(c)$ and $p(c)$ related?
2. Use the definition of $G(\cdot)$ to write the optimization problem faced by each firm. Define an equilibrium.
3. Assume that $G(\cdot)$ does not have holes and is atomless. Use the first order condition associated with the firms' optimization problem and derive a differential equation that $G(\cdot)$ must satisfy. What is the boundary condition that $G(\cdot)$ must satisfy?
4. Solve the differential equation for $G(\cdot)$. What does the distribution of prices look like qualitatively? See the paper by Ellison and Ellion, "MATCH QUALITY, SEARCH, AND THE INTERNET MARKET FOR USED BOOKS" for some evidence on price dispersion in the used book markets. How does their evidence compare to the price distribution in this model?

Problem 4. On the Job Search and Wage Posting

Here, you extend the Burdett and Judd model to allow for dynamics and on the job search.

Consider a model consisted of a continuum of workers and firms – both unit continuum. For now, let us assume that wages are distributed according to $F(w)$ which we will derive later. Time is continuous and workers discount the future at rate ρ. Throughout the question, we assume that the model is in steady state and therefore, wage disperson, unemployment, etc. are independent of time.

Suppose that both employed and unemployed workers find a wage offer at rate λ, i.e., arrival of the offer is expotentially distributed with parameter λ. Note that this parameter is the same for both employed and unemployed workers. If the worker is employed, she simply takes the offer if the new wage is higher than her current wage. Assume that workers get fired at rate δ and that wages are constant over the period of employment. Moreover, the flow value of unemployment is b. Let u be the unemployment rate.

1. Write the Bellman equations for the employed and unemployed workers. Let U be the value of the unemployed and $V(w)$ be the value of the employed worker with wage w. What is an unemployed worker's reservation wage?

2. Let $G(w)$ be the distribution of wage among the workers. Note that this is not necessarily the same as $F(\cdot)$ the distribution of wages since workers search on the job and "climb up the job ladder". In other words, $G(\cdot)$ is the economy-wide distribution of wages among the workers while $F(\cdot)$ is the distribution of wages faced by each individual employed and unemployd worker. How does $G(\cdot)$ relate to $F(\cdot)$? To calculate this relationship, you should think about the change in the measure of workers whose wage is below w. What is the inflow? What is the outflow? In a steady state, the inflow must be equal to the outflow for all wages offered in equilibrium. What is the steady state unemployment rate?

3. Write down the payoff for a firm that offers a wage w. Assume that the output of the firm once it employs a worker is z and the firm also discount future payoffs at rate ρ. *Hint: The measure of workers met by a firm that sets a wage* w *is* $u + (1-u)G(w)$. *Why?*

4. In equilibrium, as in Burdett and Judd, firms must be indifferent between wages offered in equilibrium. Given this indifference condition solve for $F(\cdot)$. What is the lowest wage offered in equilibrium?

5. Solve for the distribution of wages among workers $G(w)$. Does it look like what we observe in the data?