Problem Set 2
Macro 2 - Spring 2020

Problem 1. Computation of the Huggett Model

Consider the Huggett model discussed in class. Suppose that log income takes on values in $\{-0.1, 0, 0.1\}$, i.e, $[\log y_1, \log y_2, \log y_3] = [-0.1, 0, 0.1]$. Suppose that the transition matrix is given by

$$P = \begin{bmatrix} 0.75 + \frac{0.25}{3} & \frac{0.25}{3} & \frac{0.25}{3} \\ \frac{0.25}{3} & 0.75 + \frac{0.25}{3} & \frac{0.25}{3} \\ \frac{0.25}{3} & \frac{0.25}{3} & 0.75 + \frac{0.25}{3} \end{bmatrix}$$

where P_{ij} is the probability of state i at $t+1$ if state is j at t. Additionally, suppose that individual utility is given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \log c_t$$

Assume that a period is a year and set β to 0.9. Suppose that the lower bound on assets is $\underline{a} = 0.3$. Specify a grid for r between -1 and 0.1.

1. For each value of r do the following:
   (a) Solve the income fluctuation problem using value function iteration.
   (b) Solve the income fluctuation problem using policy function iteration and a discretization of the asset space with a 1000 point grid.
   (c) Solve the income fluctuation problem by finding a policy function that solves the Euler equation using a linear approximation and the endogenous grid method. Your grid should have 100 points.
   (d) Using the solution from either of these methods, solve for the stationary distribution of assets and income.
2. For one value of r, plot the policy functions that you solved for using each method.
3. Report average computation time for each method.
4. Find the market clearing interest rate.

Problem 2. Hugget meets McCall.

Consider an economy populated by a continuum of workers that are either employed or unemployed. Time is discrete and is given by $t = 0, 1, \cdots$. If employed a worker's wage is given by $w_t \in [\underline{w}, \overline{w}]$ which is constant as long as they are employed; in each period, the probability of losing their job is equal to λ'. An unemployed worker receives a job offer in each period with probability λ and then draw a wage from a distribution $F(w)$; at the end of the period, if a worker is unemployed they collect unemployment benefit given by b.

Unlike the McCall model, we assume that workers can borrow and save at rate r while their preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Like Huggett's model, assume that there is a borrowing constraint of the form $a_{t+1} \geq \underline{a}$ with $\underline{a} \leq 0$. Furthermore, assume that r is determined in equilibrium so that total borrowing and saving in this economy is zero.

1. Define a recursive competitive equilibrium for this economy. You may assume that the economy is in Steady State.

2. How does adding assets change the main insights from the McCall model? In particular, describe what happens to reservation wages. Provide an equation that describes how reservation wages are determined. You may assume that interest rate is lower than the discount rate $1/\beta - 1$.

3. Suppose that $\underline{a} = 0$, solve for the equilibrium allocations. *Hint: Assets holdings are easily calculated.*

4. Suppose that $\underline{a} = 0$. Solve for the market clearing interest rate. *Hint: There are multiple market clearing interest rates.*

5. In this part, you have to numerically solve for equilibrium allocation assuming certain numerical values which are described below. Note that in what follows, I have normalized average offers to 1[1] and have adjusted unemployment benefit and borrowing limits in relation with this normalization:

   (a) Assume that annual discount rate is aroung 4%, i.e., $\beta = 0.96$.

   (b) See the paper by Hornstein, Krusell, Violante (2011, AER) for values of λ and λ'.

   (c) Assume that offers have a log-normal distribution so that

   $$\log w \sim \mathcal{N}(0, \sigma)$$

   and use two values for $\sigma = 0.2$ and 0.4.

   (d) Finally assume that $u(c) = \log c$, that $b = 0.3$ and that $\underline{a} = -0.3$.

   Given this description, solve for the equilibrium allocations, i.e., consumption, saving, and reservation wages together with equilibrium interest rate assuming that the economy is in steady-state. Note that in order to do this, you have to discretize the offer distribution. You may use the Tauchen-Hussey method to do that – look it up! As to solve the dynamic programming problem, I recommend one of three ways: 1. value function iteration; 2. policy function iteration, or 3. endogenous grid. Endogenous grid is the fastest method but you must familiarize yourself with it.

6. In your solution above, compare the equilibrium wage distribution to an economy where $\underline{a} = 0$. Do you think allowing for borrowing and lending can overcome the Hornstein, Krusell and Violante puzzle?

7. Increase unemployment benefit to 0.4. How does the search behavior of workers change over their unemployment spell?

Problem 3. A Model with Idiosyncratic Rate of Return Shocks.

Consider a model with cotinuum of individuals who are subject to idiosyncratic returns on their investment. In particular, each individual can invest in a risk free bond with rate of return r and a risky capital whose rate of return is random and variable across individuals. More specifically, for one unit

[1] More formally, offer distribution has a long-normal distribution with average log value of 0. If st. dev. of offer distribution is σ, then the its mean value is $e^{\frac{1}{2}\sigma^2}$ which is fairly close to 1.

of investment at time t in the risky capital, the consumer earns an income of z_{t+1} – in addition to the principal. We assume that $z_{t+1} = z_t$ with probability λ and is drawn from some distribution $F(z)$ with probability $1 - \lambda$. Assume that individuals can borrow using the risky free bond while their investment in risky capital has to be positive.

Suppose that individuals have preferences given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \log c_t$$

where σ is relative risk aversion.

1. Write the budget constraint of each individual.
2. Write the optimization problem faced by each individul in both sequence form and recursively. What is the state variable.
3. Solve the policy function of the individual using the guess and verify method.
4. Now suppose that we have a closed economy consisted of individuals like above who are symmetric and can trade the risk-free bond. Define an equilibrium in this economy.
5. For a given risk-free rate, write an equation that describes the stationary distribution of the relevant state variable.
6. Suppose that there is no risk-free bond so their choice is how much to spend on consumption or risky capital. Moreover, supppose that $\lambda = 0$. Suppose that a stationary distribution of wealth exists. Show that it must be a pareto distribution.
7. Show that for the example above, a stationary distribution of wealth does not exist!