Problem Set 3
Macro 2 - Spring 2020

Problem 1. Disability Insurance

Consider the model of risk sharing studied in class but suppose that individuals face disability shocks, i.e., suppose that the face shocks that makes them disable forever. In other words, income takes two values 0 and y and the transition probability is given by

$$\pi\left(y_{t+1}|y_t\right)=\begin{cases}\lambda & y_{t+1}=y, y_t=y\\ 1-\lambda & y_{t+1}=0, y_t=y\\ 1 & y_{t+1}=0, y_t=0\end{cases}$$

The goal is to study optimal provision of disability insurance given that it is not very difficult to fake disability.

1. Write down the problem of a planner that maximizes ex-ante utility and observes income of all individuals.
2. What can you say about optimal consumption in the solution to the above planning problem.
3. Now suppose that individuals can fake disability, describe in words how the allocation in part 2 does not satisfy incentive compatibility similar to what we described in class. In other words, facing the allocations described in part 2, how do individuals fake disability. Note that in class, effort was not observable. Here, income is not.
4. Write a version of incentive compatibility constraint assuming that we want everyone to tell the truth about their disability status.
5. Derive the one-shot incentive compatibility constraint.
6. Write down the planning problem and its dual.
7. Formula the problem recursively.
8. What can you say about the optimal allocations? You may assume that interest rates are the inverse of the discount rate? Describe intuitively what type of policies implement optimal allocations.

Problem 2. Trade and Inequality

Consider the DFS model with the following change: suppose that workers are heterogeneous with respect to their skills. In particular, in each country there is a worker skill distribution $F\left(s\right)$ and $F^*\left(s\right)$ – respectively for home and foreign country. Suppose that production function for producing variety ω is given by $\int G\left(a\left(\omega\right), s\right) L\left(s\right) ds$ where $L\left(s\right)$ is the number of workers with skill s – this is the same in both countries – where we assume that

$$\frac{\partial}{\partial s \partial a}\log G\left(a, s\right) > 0$$

This property is also called strict log super-modularity. Note that firms can hire any number of workers of any type.

As in the DFS model $a(\omega)$ is increasing and $a^*(\omega)$ decreasing. Firms can choose any type of worker to employ. The strict log supermodularity assumption guarantees that each type of firm employs only one type of worker and firms with higher productivity (higher $a(\omega)$ or $a^*(\omega)$) hire more productive workers (workers with higher s). This is called the assortative matching property. Suppose that the function that describes how workers match to firms is $M(s)$ ($M^*(s)$ for the foreign country)

1. Suppose that each country is in autarky. Define a competitive equilibrium of this economy – do this only for home.

2. Suppose that matching function is given by $M(s)$. What is the density of distribution of output? That is, what is the output of firms with productivity $\omega = M(s)$?

3. Describe the optimality conditions for the firms in each country as well as market clearing. Your answer should be two differential equations!

4. Now suppose that trade is free between the two countries. Define a competitive equilibrium.

5. Rewrite the equilibrium conditions of part 2 for the world economy.

6. Now suppose that $G(a, s) = e^{a \cdot s}$, and the p.d.f. of s, $f(s) = Ce^{-s-s^2}$ has a truncated normal distribution for $s \in [1, 2]$ where C is an appropriate constant. Suppose that $a(\omega) = \omega$ and $a^*(\omega) = 1 - \omega$. Solve for equilibrium allocations under autarky and free trade and describe what happens to inequality.