Problem Set 4

Problem 1. Equivalent Taxes   Consider the one sector growth model – the version where consumption and investment goods are produced with the same constant returns to scale technology. Suppose that households value leisure and labor supply is elastic. Consider the following two tax schedules:

1. A constant tax rate on consumption by households.

2. A constant tax rate on capital income which allows for deduction of investment from the tax base.

Assume that government has access to debt. Moreover, the taxes in both systems are used to finance the same stream of government purchases.

a. Define the tax distorted competitive equilibrium for each of the tax systems.

b. Describe under each tax schedule which margings are distorted.

c. Are these tax schedules equivalent?

d. Which of these tax schedules are equivalent to a lump-sum tax? Prove your result.

e. Consider the steady state level of capital under either tax system. Calculate the level of capital under each tax schedule. Which one is higher?

Problem 2. Mortgage Deduction.   In this problem, we consider the effect of mortgage interest deduction on macroeconomic variables. Modelling mortgages requires thinking about a model with savers and borrowers which is more complicated. Here, we take a shortcut in thinking about it.

   Consider the one-sector growth model with the addition that consumer also consume housing services. Suppose that preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t, s_t\right) - v\left(\ell_t\right) \right]$$

where s_t is the flow housing services. We assume that $u\left(\cdot, \cdot\right)$ is strictly concave and strictly increasing in (c, s) while $v\left(\ell\right)$ is strictly convex and increasing with $v'\left(0\right) = 0$.

   The housing stock for each household is given by h_t and $s_t = h_t$. Any changes to housing stock, investment in housing, can be done using the single produced good. In particular, housing stock changes according to

$$h_{t+1} = h_t \left(1 - \delta\right) + x_{h,t}$$

where $x_{h,t}$ is invesment in housing pruchased by housholds. Suppose that households pay labor income taxes – at rate $\tau_{l,t}$ – and housing investments can be deducted from their taxes at rate α. In other words, for every dollar spent on investment in housing, the households receive a $\alpha \tau_{l,t}$ dollar subsidy from the government. Suppose also that the government imposes a tax rate $\tau_{k,t}$ on capital income – income that households earn from renting out their capital to firms. The government has a stream of purchases that it needs to finance given by $\{g_t\}_{t=0}^{\infty}$.

a. Define a TDCE for this economy. Write the budget constraint of the households assuming sequential trading.

b. Write the optimality condition of the households in any TDCE.

c. Suppose that the economy is in steady state – assuming that labor and capital income taxes converge to constant levels τ_l and τ_k in the long-run. Provide a set of equations that describe consumption, labor supply, housing and capital stock in this steady state.

d. Consider an increase in mortgage deduction rate, α. How does this affect steady state level of capital and housing stock.

e. Now, we want to characterize optimal taxes for this economy. Derive the implementability constraint for this economy.

f. Write down the optimal taxation problem.

g. Characterize optimal capital and labor income taxes as well as optimal mortgage deduction in the long-run. Note that it might be optimal to have an additional tax on housing investment.

Problem 3. Workers and Capitalists. In this problem, we consider optimal taxation in a model with heterogeneity in access to capital. In particular, suppose that the population is consisted of capitalists and workers – the fraction of capitalists in the economy is μ while that of workers is $1-\mu$. Suppose that capitalists own all the capital – they are identical – and hire workers. Suppose that production function of the technology operated by capitalists is given by $F(k_t, A_t n_t) = k_t^\alpha (A_t n_t)^{1-\alpha}$ with $A_t = (1+g)^t$ for $g > 0$. Workers are identical and supply labor inelastically. Both workers and capitalist have preferences of the form

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

with $\beta(1+g)^{1-\sigma} < 1$. If $\sigma = 1$, then preferences are given by $\sum_{t=0}^{\infty} \beta^t \log c_t$.

Suppose that government imposes a linear tax on capitalists income – their proceeds from operating the technolygy less wages paid to the workers they hire – given by τ_{kt}. The government uses the proceeds to make transfers to the workers. Assume that the government has access to debt so that the proceeds from income taxation are not necessarily equal to the transfers to the workers. For simplicity, assume that there is no other government spending.

a. Define a TDCE for this economy.

b. Derive the implementability condition. Note that implementability condition is a condition that the TDCE imposes on allocations – beyond feasibility constraint that you derived in part a.

c. Suppose that the government's objective is to maximize the welfare of the workers. Formulate the optimal taxation problem.

d. What can you say about optimal capital income taxes in the long-run? Do they depend on the growth rate of the economy?

Problem 4. Workers and Capitalists again! Consider the setting in problem 3 but now assume that $A_t = B \int_0^1 k_t^i di$ – where capitalists are indexed by $i \in [0,1]$. Note that since capitalists are symmetric, they will all choose the same level of capital k_t. Repeat parts a-d in problem 3.

e. What can you say about optimal capital income taxes in the long-run? How do they vary with B and the growth rate of the economy?