Problem Set 5

Problem 1. Computations 1
Consider the one-sector growth model and suppose that $u(c,\ell) = \log c$ and that $F(k,n) = k^\alpha n^{1-\alpha}$. Write a code in MATLAB to solve the one-sector growth model using value function iterations. Assume that $\alpha = 1/3$, $\delta = 0.08$, and $\beta = 0.96$. Using your solution calculate the steady state level of capital. Using your computations and by referring to the notes, calculate the rate of convergence of this economy as in Barro and Sala-i-Martin's exercise.

Problem 2. Computations 2
Consider the stochastic one-sector growth model and suppose that $u(c,\ell) = \log c + \psi \log \ell$ and that $y_t = A_t k_t^\alpha n_t^{1-\alpha}$. Assume that $\alpha = 1/3$, $\beta = 0.96$ and $\delta = 0.08$.

a. If we were to calibrate ψ so that households on average work, 1/3 of their time, what value do you get for ψ by matching this statistic in the non-stochastic steady state of the model.

b. Suppose that A_t follows a Markov process which is described as follows:

$$A_t \in \{A_L = 0.9, A_H = 1.1\}$$

with the transition matrix given by

$$P = \begin{bmatrix} P_{HH} & P_{HL} \\ P_{LH} & P_{LL} \end{bmatrix} = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

where $P_{ij} = \Pr(A_{t+1} = A_i | A_t = A_j)$. Solve the model using value function iteration in MATLAB.

c. Calculate the stationary distribution associated with the policy functions that you found in part b.

d. Using the stationary distribution that you found in part c, calculate standard deviation of GDP, consumption, investment, and hours. Discuss the results.

Problem 3. The Merton Problem
Consider a consumer who has standard CRRA utility function

$$\sum_{t=0}^{T} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

and lives for T periods. Suppose that in each period the consumer can invest in a risk-free asset with gross return R_f and a risky assets whose gross return is given by R_t which is an i.i.d. random variable distributed according to some continuous distribution $F(R)$. Suppose that initially, the consumer's wealth is given by W_0.

a. Write down the sequence of budget constraints for the consumer.

b. Formulate the consumer's problem recursively.

c. Solve the consumer's problem using a guess and verify method.

d. From part c, what does the portfolio choice of the consumer look like, i.e., what fraction of his/her investments in each period are in bonds vs stocks? How does this depend on age/time? Is this in line with the advice given by financial planners? If not, what ingredients can be added to the model to address this discrepancy?

Problem 4.
Solve problems 14.2, 14.3, 14.4, 14.8, 14.9, 14.10.

Problem 5. McCall meets DMP Consider the standard DMP model discussed in class with the following modification: when a worker and a firm meet they draw a match specific productivity, z from a continuous distribution $G(z)$. Suppose that z has a pareto distribution given by

$$G(z) = 1 - \left(\frac{z}{z_0}\right)^{-\gamma}$$

This productivity lasts for the duration of the job - until separation. The rest is identical to the model in class - separation rate is independent of the job productivity. Suppose that for each level of productivity that is accepted by firms and workers, there is an equilibrium wage $w(z)$. Assume that this wage is determined via Nash bargaining between the firm and the worker.

a. Write the Bellman equations for the workers and firms as a function of the wage function $w(z)$.

b. Characterize the value of reservation wage.

c. Derive the steady-state equilibrium conditions.

d. How does the wage distribution depend on unemployment benefits?

Problem 6. Solve problems 6.7, 6.8, 6.10, 6.18 in Ljungqvist and Sargent.