Notes on Postel-Vinay and Robin (2002)

Postel-Vinay and Robin (PVR) develop a fairly tractable model to explain residual wage inequality. This is an alternative to Burdett and Mortensen which is a model of wage posting and somewhat similar to Burdett and Judd that we mentioned in class. Here I describe the model and summarize its theoretical results:

- The model is consisted of a mass M of workers and a unit continuum of firms
- Workers are heterogeneous and their type, i.e., labor productivity is given by ε which we assume that has a distribution $H(\varepsilon)$ with density $h(\varepsilon)$ – assume that a density exists everywhere. Moreover, $Supp(H) = [\varepsilon_{min}, \varepsilon_{max}]$.
- Firms are heterogeneous and their type is given by p with cdf given by $\Gamma(p)$ and pdf given by $\gamma(p)$. If a worker of type ε works for a firm of type p, the output is εp. $Supp(\Gamma) = [p_{min}, p_{max}]$
- Time is continuous and unemployed workers find jobs at rate λ_0. Furthermore, workers find jobs at rate λ_1 while they separate from their jobs at rate δ. Additionally, a worker (employed or unemployed) that finds a new match, draws a firm with productivity p distributed according to cdf $F(p)$ – with pdf $f(p)$.
- Workers' discount factors are given by ρ and they have period utility function given by $U(x)$. Moreover, the flow value of unemployment is given by $b\varepsilon$ for some $b < p_{min}$.
- Everything is observable to all parties invoved: firms and workers. When a worker that works at a firm with productivity p matches with a firm p', the two firms compete over the worker by playing a limit-pricing game wherein the more productive firm is able to outbid the less productive one and the wages are determined to ensure that the profits of the less productive firm is 0. We formalize this below.
- Formally, we can define two value functions for workers: $V_0(\varepsilon)$ for an unemployed worker of type ε and $V(\varepsilon, w, p)$ for a worker of type ε that works for p. Obviously V is increasing in all of its elements.
- We also define the following functions: $\phi_0(\varepsilon, p)$, $\phi(\varepsilon, p, p')$

$$V_0(\varepsilon) = V(\varepsilon, \phi_0(\varepsilon, p), p)$$
$$V(\varepsilon, \phi(\varepsilon, p, p'), p') = V(\varepsilon, \varepsilon p, p)$$

In words, ϕ_0 is the wage that makes the worker indifferent between unemployment and having a job at firm p. Since firms have all the bargaining power, i.e., they make take-it-or-leave-it offer, this is the wage that will be offered by firm p to an unemployed worker. $\phi(\varepsilon, p, p')$ is the lowest wage that firm $p' > p$ can offer that makes the worker want to switch from firm p. The RHS of the above equality is the value for a worker of type ε at a firm of type p that pays the worker the maximum possible wage εp, i.e., all the output produced at p. That is if firm p' offers any wage $w' > \phi(\varepsilon, p, p')$, firm p is not able to keep the worker. Under the limit pricing assumption, in this situation the worker switches from p to p' and the wage at p' is given by $\phi(\varepsilon, p, p')$. Therefore, for a worker that currently works at firm p for wage w and matches with a firm p', there are three possibilities:

1. $p' > p$: In this case, the worker quits her current job and switches to p'. The wage upon switching will be $\phi(\varepsilon, p, p')$
2. $q(\varepsilon, w, p) \leq p' \leq p$ where $q(\cdot, \cdot, \cdot)$ is defined by
$$\phi(\varepsilon, q(\varepsilon, w, p), p) = w$$
or
$$V(\varepsilon, w, p) = V(\varepsilon, \varepsilon q(\varepsilon, w, p), q(\varepsilon, w, p))$$
In other words, $q(\varepsilon, w, p)$ is the lowest productivity that can offer a wage at least equal to w and break-even. Note that a firm with productivity lower than $q(\varepsilon, w, p)$ is not productive enough to be able to pay w. In this case when $p' \in [q, p]$, the worker stays at the firm yet she will get a raise and her wage increases to $\phi(\varepsilon, p', p)$.
3. $p' < q(\varepsilon, w, p)$: In this case, the wage stays unchanged and the worker does not move.

- We can use the above discussion to fully characterize all value functions and wages. To see this, note that we have
$$\begin{aligned}\rho V_0(\varepsilon) &= U(b\varepsilon) + \lambda_0 \int [V(\varepsilon, \phi_0(\varepsilon, p), p) - V_0(\varepsilon)] \, dF(p) \\ &= U(b\varepsilon) \rightarrow V_0(\varepsilon) = \frac{U(b\varepsilon)}{\rho}\end{aligned}$$
This is because the firms set wages so that an unemployed worker is just indifferent between employment and unemployment. As a result there is no option value of searching for the unemployed.

- Vale function for the workers
$$\begin{aligned}\rho V(\varepsilon, w, p) =& U(w) + \lambda_1 \int_{q(\varepsilon,w,p)}^{p} [V(\varepsilon, \varepsilon p', p') - V(\varepsilon, w, p)] \, dF(p') + \\ & \lambda_1 \int_{p}^{p_{max}} [V(\varepsilon, \varepsilon p, p) - V(\varepsilon, w, p)] \, dF(p') + \\ & \lambda_0 [V_0(\varepsilon) - V(\varepsilon, w, p)]\end{aligned}$$
Note that working for a firm of type p has an option value. It comes with an option of future wage increases. Now we use the above results about wage setting to solve for the value functions and wages:

- Set $w = \varepsilon p$. Intuitively, a worker at p with this wage is at the top of the wage ladder in this firm so any offer from someone else with $p' < p$ will be rejected by the worker. Morevoer, any firm with $p' > p$ will make an offer to the worker so that her valuation remains unchanged. So we get the following:

$$(\rho+\delta) V(\varepsilon, \varepsilon p, p) = U(\varepsilon p) + \delta V_0(\varepsilon) \to V(\varepsilon, \varepsilon p, p) = \frac{U(\varepsilon p) + \delta V_0(\varepsilon)}{\rho+\delta}$$

  We then can write

$$\begin{aligned}(\rho+\delta) V(\varepsilon, w, p) =& U(w) + \delta V_0(\varepsilon) \\ & - \lambda_1 \int_q^p \left[V(\varepsilon, \varepsilon p', p') - V(\varepsilon, w, p)\right] d\left(1 - F(p')\right) \\ & + \lambda_1 \left[V(\varepsilon, \varepsilon p, p) - V(\varepsilon, w, p)\right]\left(1 - F(p)\right) \\ =& U(w) + \delta V_0(\varepsilon) \\ & - \lambda_1 \left[V(\varepsilon, \varepsilon p', p') - V(\varepsilon, w, p)\right]\left(1 - F(p')\right)\Big|_q^p \\ & + \lambda_1 \int_q^p \left(1 - F(p')\right) \frac{\varepsilon U'(\varepsilon p')}{\rho+\delta} dp' \\ & + \lambda_1 \left[V(\varepsilon, \varepsilon p, p) - V(\varepsilon, w, p)\right]\left(1 - F(p)\right) \\ =& U(w) + \delta V_0(\varepsilon) \\ & + \lambda_1 \int_q^p \left(1 - F(p')\right) \frac{\varepsilon U'(\varepsilon p')}{\rho+\delta} dp'\end{aligned}$$

  Now we can use the definition of ϕ adn write

$$\begin{aligned}(\rho+\delta) V(\varepsilon, \varepsilon\hat{p}, \hat{p}) &= U(\phi(\varepsilon, \hat{p}, p)) + \delta V_0(\varepsilon) + \lambda_1 \int_{\hat{p}}^p \left(1 - F(x)\right) \frac{\varepsilon U'(\varepsilon x)}{\rho+\delta} dx \\ U(\varepsilon\hat{p}) &= U(\phi(\varepsilon, \hat{p}, p)) + \lambda_1 \int_{\hat{p}}^p \left(1 - F(x)\right) \frac{\varepsilon U'(\varepsilon x)}{\rho+\delta} dx \\ U(\phi(\varepsilon, \hat{p}, p)) &= U(\varepsilon\hat{p}) - \lambda_1 \int_{\hat{p}}^p \left(1 - F(x)\right) \frac{\varepsilon U'(\varepsilon x)}{\rho+\delta} dx\end{aligned}$$

- This is the key equation of the paper that determines someone wage. In other words, we can say

$$w_t = \phi(\varepsilon, \hat{p}_t, p_t)$$

  where p_t is the current productivity of a worker's employer and $\hat{p}_t$ is the last most productive firm to make her an offer. Moreover, when $U(x) = \frac{x^{1-\alpha}-1}{1-\alpha}$ or $U(x) = \log x$, we can write the above as

$$\begin{aligned}\log\phi(\varepsilon, \hat{p}, p) &= \log\varepsilon + \frac{1}{1-\alpha}\log\left[\hat{p} - \frac{\lambda_1}{\rho+\delta}\int_{\hat{p}}^p \left(1 - F(x)\right) x^{-\alpha} dx\right], \quad \alpha \neq 1 \\ \log\phi(\varepsilon, \hat{p}, p) &= \log\varepsilon + \log\hat{p} - \frac{\lambda_1}{\rho+\delta}\int_{\hat{p}}^p \frac{1 - F(x)}{x} dx\end{aligned}$$

One thing to notice is that the wage is below $\varepsilon\hat{p}$ exactly because of the option value. We also see that wage profiles are concave and this is because as workers move up the job ladder, the option values decline. Quite intuitively, the higher is the rate of getting an offer on the job λ_1 the higher is this option value and the lower is the wage.

- Steady State: In order to take the model to the data, we need to impose some discipline on the aggregates and the easiest way of doing this would be to assume stationarity. Then, we have

  - unemployment rate

$$(1-u)\,\delta - \lambda_0 u = 0 \to u = \frac{\delta}{\delta+\lambda_0}$$

  - Number of workers at firms p: cdf: $L(p)$

$$-L(p)\left[\delta+\lambda_1\left(1-F(p)\right)\right](1-u)+uF(p)\,\lambda_0 = 0 \to L(p) = \frac{\delta F(p)}{\delta+\lambda_1\left(1-F(p)\right)}$$

  - Number of workers of type ε at p: pdf: $\ell(\varepsilon,p)$ given by

$$\ell(\varepsilon,p) = h(\varepsilon)\,\ell(p) = h(\varepsilon)\,L'(p) = \frac{\delta\left(\delta+\lambda_1\right)f(p)}{\left(\delta+\lambda_1\left(1-F(p)\right)\right)^2}$$

  - Distribution of wages conditional on (ε,p): cdf: $G(w|\varepsilon,p)$

$$-G(w|\varepsilon,p)\,\ell(p)\,h(\varepsilon)\,(1-u)\left(\delta+\lambda_1\left(1-F\left(q\left(\varepsilon,w,p\right)\right)\right)\right)$$

$$+\lambda_0 u h(\varepsilon)\,f(p)+\lambda_1 f(p)\,(1-u)\,h(\varepsilon)\int_{p_{min}}^{q(\varepsilon,w,p)}\ell(p')\,dp' = 0$$

$$-G(w|\varepsilon,p)\,\frac{\delta\left(\delta+\lambda_1\right)\left(\delta+\lambda_1\left(1-F(q)\right)\right)}{\left(\delta+\lambda_1\left(1-F(p)\right)\right)^2}$$

$$+\delta+\lambda_1\frac{\delta F(q)}{\delta+\lambda_1\left(1-F(q)\right)} = 0$$

$$\frac{\left(\delta+\lambda_1\left(1-F(p)\right)\right)^2}{\left(\delta+\lambda_1\left(1-F(q)\right)\right)^2} = G(w|\varepsilon,p)$$

  Note that since $b < p_{min}$, there is a flat in $G(w|\varepsilon,p)$. We can also write the above as

$$G\left(\phi\left(\varepsilon,\hat{p},p\right)|\varepsilon,p\right) = \frac{\left(\delta+\lambda_1\left(1-F(p)\right)\right)^2}{\left(\delta+\lambda_1\left(1-F(\hat{p})\right)\right)^2}$$

- I am going to leave the econometrics out but the authors structurally estimate this model – which is quite difficult given the nature of the data they have. The main result is that person effects explain only a small fraction of the variation in wages.